$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 \leqslant r^2$

$\overrightarrow{P_1P_2} = (x_2 - x_1)\vec{\imath} + (y_2 - y_1)\vec{\jmath} + (z_2 - z_1)\vec{k}$

$\overrightarrow{P_1P_3} = (x_3 - x_1)\vec{\imath} + (y_3 - y_1)\vec{\jmath} + (z_3 - z_1)\vec{k}$

$\overrightarrow{P_1P} = (x - x_1)\vec{\imath} + (y - y_1)\vec{\jmath} + (z - z_1)\vec{k}$

$$\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$

$\vec{n} = [(y_2 - y_1)(z_3 - z_1) - (y_3 - y_1)(z_2 - z_1)]\vec{\imath} - [(x_2 - x_1)(z_3 - z_1) - (x_3 - x_1)(z_2 - z_1)]\vec{\jmath}$
$+ [(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)]\vec{k}$

$\overrightarrow{P_1P} = (x - x_1)\vec{\imath} + (y - y_1)\vec{\jmath} + (z - z_1)\vec{k}$

$\vec{n} \cdot \overrightarrow{P_1P} = 0$

$$\overbrace{[(y_2 - y_1)(z_3 - z_1) - (y_3 - y_1)(z_2 - z_1)]}^{\alpha}(x - x_1) - \overbrace{[(x_2 - x_1)(z_3 - z_1) - (x_3 - x_1)(z_2 - z_1)]}^{\beta}(y - y_1) +$$
$$\overbrace{[(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)]}^{\gamma}(z - z_1) = 0$$

$\alpha(x - x_1) + \beta(y - y_1) + \gamma(z - z_1) = 0$

$\alpha x + \beta y + \gamma z + \underbrace{(-\alpha x_1 - \beta y_1 - \gamma z_1)}_{-d} = 0$

$\alpha x + \beta y + \gamma z - d = 0 \quad \Rightarrow \boxed{\alpha x + \beta y + \gamma z = d}$

$\alpha = a \;\; ; \;\; \beta = b \;\; ; \;\; \gamma = c \;\; \Rightarrow \boxed{ax + by + cz = d}$